因为csdn审核太慢了，所以发个pdf的版本，csdn网址(以后错误修改直接更新到csdn上)：
http://blog.csdn.net/u013004597/article/details/52069741
在看本博客之前请先看svo的Supplementary matterial .以下的过程借鉴了肖师兄的depth filter.pdf和高翔的推导，将给出公式(17)-(26)的推导推导过程，如果发现有错误之处，敬请指正.(qq:1347893477 孙志明-东北大学)

$$
\begin{aligned}
&(\pi N(x|Z,\tau^2)) + (1-\pi)U(x))N(Z|\mu,\sigma^2)Beta(\pi|a,b) && (17)\\
=&\pi N(x|Z,\tau^2)N(Z|\mu,\sigma^2)Beta(\pi|a,b) + (1-\pi)U(x)N(Z|\mu,\sigma^2)Beta(\pi|a,b)\\
=&\frac{a}{a+b}N(Z|\mu,\sigma^2)N(Z|\mu,\sigma^2)Beta(\pi|a+1,b) + \frac{b}{a+b}U(x)N(Z|\mu,\sigma^2)Beta(\pi|a,b+1)\\
=&\frac{a}{a+b}N(x|\mu,\sigma^2+\tau^2)N(Z|m,s^2)Beta(\pi|a+1,b) + \frac{b}{a+b}U(x)N(Z|\mu,\sigma^2)Beta(\pi|a,b+1) && (18)
\end{aligned}
$$

对比可知：$N(x|Z,\tau^2)N(x|Z,\sigma^2) = N(x|\mu,\sigma^2+\tau^2)N(Z|m,s^2)$
取出$N(x|Z,\tau^2)N(x|Z,\sigma^2)$展开为：

$$
\begin{aligned}
&N(x|Z,\tau^2)N(Z|\mu,\sigma^2)\\
=&\frac{1}{\sqrt{2\pi}\tau}e^{-\frac{(x-Z)^2}{2\tau^2}}\cdot\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(Z-\mu)^2}{2\sigma^2}}\\
=&\frac{1}{2\pi\tau\sigma}e^{-\frac{(x-Z)^2}{2\tau^2}-\frac{(Z-\mu)^2}{2\sigma^2}}\\
=&\frac{1}{2\pi\tau\sigma}e^{-\frac{\sigma^2(x-Z)^2+\tau^2(Z-\mu)^2}{2\tau^2\sigma^2}}\\
=&\frac{1}{2\pi\tau\sigma}e^{-\frac{(\sigma^2+\tau^2)Z^2-2(x\sigma^2+\mu\tau^2)Z+x^2\sigma^2+\mu^2\tau^2}{2\tau^2\sigma^2}}\\
=&\frac{1}{\sqrt{2\pi(\sigma^2+\tau^2)}}\frac{1}{\sqrt{2\pi\frac{\sigma^2\tau^2}{(\sigma^2+\tau^2)}}}e^{-\frac{(x-\mu)^2}{2(\sigma^2+\tau^2)}}\cdot e^{-\frac{(\sigma^2+\tau^2)Z^2-2(x\sigma^2+\mu\tau^2)Z+x^2\sigma^2+\mu^2\tau^2}{2\tau^2\sigma^2}+\frac{(x-\mu)^2}{2(\sigma^2+\tau^2)}}\\
=&\frac{1}{\sqrt{2\pi(\sigma^2+\tau^2)}}e^{-\frac{(x-\mu)^2}{2(\sigma^2+\tau^2)}}\cdot\frac{1}{\sqrt{2\pi\frac{\sigma^2\tau^2}{(\sigma^2+\tau^2)}}}e^{-\frac{((\sigma^2+\tau^2)Z^2-2(x\sigma^2+\mu\tau^2)Z+x^2\sigma^2+\mu^2\tau^2)(\sigma^2+\tau^2)+(x-\mu)^2\tau^2\sigma^2}{2\tau^2\sigma^2(\sigma^2+\tau^2)}}\\
=&N(x|\mu,\sigma^2+\tau^2)\cdot\frac{1}{\sqrt{2\pi\frac{\sigma^2\tau^2}{(\sigma^2+\tau^2)}}}e^{-\frac{(\sigma^2+\tau^2)^2Z^2-2(x\sigma^2+\mu\tau^2)(\sigma^2+\tau^2)Z+(x\sigma^2+\mu\tau^2)^2}{2\tau^2\sigma^2(\sigma^2+\tau^2)}}\\
=&N(x|\mu,\sigma^2+\tau^2)\cdot\frac{1}{\sqrt{2\pi\frac{\sigma^2\tau^2}{(\sigma^2+\tau^2)}}}e^{-\frac{[(\sigma^2+\tau^2)Z-(x\sigma^2+\mu\tau^2)]^2}{2\tau^2\sigma^2(\sigma^2+\tau^2)}}\\
=&N(x|\mu,\sigma^2+\tau^2)\cdot\frac{1}{\sqrt{2\pi\frac{\sigma^2\tau^2}{(\sigma^2+\tau^2)}}}e^{-\frac{[Z-\frac{(x\sigma^2+\mu\tau^2)}{\tau^2+\sigma^2}]^2}{2(\frac{\tau^2\sigma^2}{\tau^2+\sigma^2})}}\\
=&N(x|\mu,\sigma^2+\tau^2)N(Z|\frac{(x\sigma^2+\mu\tau^2)}{\tau^2+\sigma^2},\frac{\tau^2\sigma^2}{\tau^2+\sigma^2)})\\
=&N(x|\mu,\sigma^2+\tau^2)N(Z|m,s^2)
\end{aligned}
$$

所以$s^2 = \frac{\tau^2\sigma^2}{\tau^2+\sigma^2}, m = \frac{(x\sigma^2+\mu\tau^2)}{\tau^2+\sigma^2}$,进一步简化：

$$\frac{1}{s^2} = \frac{1}{\tau^2} + \frac{1}{\sigma^2} \tag{19}$$

$$\begin{aligned} m &= \frac{(x\sigma^2+\mu\tau^2)}{\tau^2+\sigma^2} \\ &= s^2(\frac{x}{\tau^2} + \frac{\mu}{\sigma^2}) \end{aligned} \tag{20}$$

现在假设:

$$C_1 = \frac{a}{a+b}N(x|\mu, \sigma^2+\tau^2) \tag{22}$$

$$C_2 = \frac{b}{a+b}U(x) \tag{20}$$

公式(16)和(18)的一阶矩和二阶矩相等，先求公式(16)的一、二阶矩：
Z的一、二阶矩：

$$\begin{aligned} \int Zp(Z,\pi|a',b',\mu',\sigma')dZd\pi &= \int ZN(Z|\mu',\sigma'^2)Beta(\pi|a',b')dZd\pi \\ &= \int ZN(Z|\mu',\sigma'^2)dZ \\ &= \mu' \end{aligned}$$

$$\begin{aligned} \int Z^2p(Z,\pi|a',b',\mu',\sigma')dZd\pi &= \int Z^2N(Z|\mu',\sigma'^2)Beta(\pi|a',b')dZd\pi \\ &= \int Z^2N(Z|\mu',\sigma'^2)dZ \\ &= \mu'^2+\sigma'^2 \end{aligned}$$

π的一、二阶矩：

$$\begin{aligned} \int \pi p(Z,\pi|a',b',\mu',\sigma')dZd\pi &= \int \pi N(Z|\mu',\sigma'^2)Beta(\pi|a',b')dZd\pi \\ &= \int \pi Beta(\pi|a',b')d\pi \\ &= \frac{a'}{a'+b'} \end{aligned}$$

$$\begin{aligned} \int \pi^2p(Z,\pi|a',b',\mu',\sigma')dZd\pi &= \int \pi^2N(Z|\mu',\sigma'^2)Beta(\pi|a',b')dZd\pi \\ &= \int \pi^2Beta(\pi|a',b')d\pi \\ &= \frac{a'(a'+1)}{(a'+b')(a'+b'+1)} \end{aligned}$$

对公式(18)的概率密度进行积分：

$$\begin{aligned} C &= \int \frac{a}{a+b}N(x|\mu,\sigma^2+\tau^2)N(Z|m,s^2)Beta(\pi|a+1,b) + \frac{b}{a+b}U(x)N(Z|\mu,\sigma^2)Beta(\pi|a,b+1)dZd\pi \\ &= \int C_1N(Z|m,s^2)Beta(\pi|a+1,b) + C_2N(Z|\mu,\sigma^2)Beta(\pi|a,b+1)dZd\pi \\ &= C_1+C_2 \end{aligned}$$

公式(18)的一、二阶矩：Z的一、二阶矩：

$$
\begin{aligned}
&\frac{1}{C}\int Z\{C_1N(Z|m,s^2)Beta(\pi|a+1,b)+C_2N(Z|\mu,\sigma^2)Beta(\pi|a,b+1)\}dZd\pi\\
=&\frac{1}{C}\int Z\{C_1N(Z|m,s^2)+C_2N(Z|\mu,\sigma^2)\}dZd\pi\\
=&\frac{1}{C}\{C_1m+C_2\mu\}
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{C}\int Z^2\{C_1N(Z|m,s^2)Beta(\pi|a+1,b)+C_2N(Z|\mu,\sigma^2)Beta(\pi|a,b+1)\}dZd\pi\\
=&\frac{1}{C}\int Z^2\{C_1N(Z|m,s^2)+C_2N(Z|\mu,\sigma^2)\}dZd\pi\\
=&\frac{1}{C}\{C_1(m^2+s^2)+C_2(\mu^2+\sigma^2)\}
\end{aligned}
$$

π的一二阶矩：

$$
\begin{aligned}
&\frac{1}{C}\int \pi\{C_1N(Z|m,s^2)Beta(\pi|a+1,b)+C_2N(Z|\mu,\sigma^2)Beta(\pi|a,b+1)\}dZd\pi\\
=&\frac{1}{C}\int \pi\{C_1Beta(\pi|a+1,b)+C_2Beta(\pi|a,b+1)\}dZd\pi\\
=&\frac{1}{C}\{C_1\frac{a+1}{a+b+1}+C_2\frac{a}{a+b+1}\}
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{C}\int \pi^2\{C_1N(Z|m,s^2)Beta(\pi|a+1,b)+C_2N(Z|\mu,\sigma^2)Beta(\pi|a,b+1)\}dZd\pi\\
=&\frac{1}{C}\int \pi^2\{C_1Beta(\pi|a+1,b)+C_2Beta(\pi|a,b+1)\}dZd\pi\\
=&\frac{1}{C}\{C_1\frac{(a+1)(a+2)}{(a+b+1)(a+b+2)}+C_2\frac{a(a+1)}{(a+b+1)(a+b+2)}\}
\end{aligned}
$$

对C_1C_2进行归一化：

$$
\begin{aligned}
C_1'&=\frac{C_1}{C}\\
C_2'&=\frac{C_2}{C}
\end{aligned}
$$

(16)(18)的阶矩相等，所以：

$$\mu'=C_1'm+C_2'\mu \tag{23}$$

$$\mu'^2+\sigma'^2=C_1'(m^2+s^2)+C_2'(\mu^2+\sigma^2) \tag{24}$$

$$\frac{a_n'}{a_n'+b_n'}=C_1'\frac{a+1}{a+b+1}+C_2'\frac{a}{a+b+1} \tag{25}$$

$$\frac{a_n'(a_n'+1)}{(a_n'+b_n')(a_n'+b_n'+1)}=C_1'\frac{(a+1)(a+2)}{(a+b+1)(a+b+2)}+C_2'\frac{a(a+1)}{(a+b+1)(a+b+2)} \tag{26}$$