Visual-Inertial Navigation: A Tutorial

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1 Notations

Table 1: **Notations**

$\mathbf{R} \in \mathbb{SO}(3)$	robot orientation in the global frame
$\mathbf{p} \in \mathbb{R}^3$	robot position in the global frame
$\mathbf{V} \in \mathbb{R}^3$	robot velocity in the global frame
$\mathbf{v} \in \mathbb{R}^3$	robot velocity in the body frame
$\mathbf{b}_a \in \mathbb{R}^3$	accelerometer bias
$\mathbf{f} \in \mathbb{R}^3$	landmark position in the global frame
$\mathbf{w}_t \in \mathbb{R}^3$	gyroscope reading at the time t
$\mathbf{a}_t \in \mathbb{R}^3$	accelerometer reading at the time t
$\mathbf{g} \in \mathbb{R}^3$	The gravity in the global frame

2 From Original Model

Here we ignore the gyroscope bias. The motion model of IMU and landmark is given in the following:

$$
\begin{aligned}
\dot{\mathbf{R}} &= \mathbf{R}S(\mathbf{w}_t) \\
\dot{\mathbf{p}} &= \mathbf{R}\mathbf{v} \\
\dot{\mathbf{v}} &= -S(\mathbf{w}_t)\mathbf{v} + (\mathbf{a}_t - \mathbf{b}_a) + \mathbf{R}^\mathsf{T}\mathbf{g} \\
\dot{\mathbf{b}}_a &= \mathbf{0} \\
\dot{\mathbf{f}} &= \mathbf{0}
\end{aligned} \tag{1}
$$

Now we define two variables $\mathbf{F}$ and $\mathbf{n}$:

$$
\begin{aligned}
\mathbf{F} &:= \mathbf{R}^\mathsf{T}(\mathbf{f} - \mathbf{p}) \\
\mathbf{n} &:= \mathbf{R}^\mathsf{T}\mathbf{g}
\end{aligned} \tag{2}
$$

According to Eq.1 and Eq.2, we get the dynamics of $\mathbf{X}$ ($\mathbf{X} = (\mathbf{F}, \mathbf{v}, \mathbf{n}, \mathbf{b}_a)$)

$$
\dot{\mathbf{X}} = \mathbf{A}_t\mathbf{X} + \mathbf{B}_t \tag{3}
$$

where $\mathbf{A}_t = \begin{bmatrix} -S(\mathbf{w}_t) & -\mathbf{I}_3 & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} \\ \mathbf{0}_{3,3} & -S(\mathbf{w}_t) & \mathbf{I}_3 & -\mathbf{I}_3 \\ \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & -S(\mathbf{w}_t) & \mathbf{0}_{3,3} \\ \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \mathbf{I}_3 \end{bmatrix}$ and $\mathbf{B}_t = \begin{bmatrix} \mathbf{0}_{3,1} \\ \mathbf{a}_t \\ \mathbf{0}_{3,1} \\ \mathbf{0}_{3,1} \end{bmatrix}$. Please note that the dynamics of $\mathbf{X}$ is linear time-varing system.

3 Observation Model of X

For simplicity, the transformation from IMU to camera is Identity. The camera measurement $\mathbf{z}_t \in \mathbb{R}^2$ for landmark at time t is

$$\mathbf{z}_t = \begin{bmatrix} u_t \\ v_t \end{bmatrix} = \mathbf{K} \begin{bmatrix} \mathbf{F}_x/\mathbf{F}_z \\ \mathbf{F}_y/\mathbf{F}_z \end{bmatrix} + \mathbf{C} \tag{4}$$

The equation above is equivalent to the following:

$$\mathbf{K}^{-1}(\mathbf{z}_t - \mathbf{C})\mathbf{F}_z = \begin{bmatrix} \mathbf{F}_x \\ \mathbf{F}_y \end{bmatrix} \tag{5}$$

i.e.,

$$\begin{bmatrix} \mathbf{I}_2 & \mathbf{K}^{-1}(\mathbf{C} - \mathbf{z}_t) \end{bmatrix} \mathbf{F} = \mathbf{0}_{2,1} \tag{6}$$

i.e.,

$$\mathbf{W}_t \mathbf{X} = \mathbf{0}_{12,1} \tag{7}$$

where

$$\mathbf{W}_t = \begin{bmatrix} \mathbf{I}_2 & \mathbf{K}^{-1}(\mathbf{C} - \mathbf{z}_t) & 0_{1,9} \end{bmatrix} \tag{8}$$

Furthermore, we can conclude a linear observation model $h(\mathbf{X}) = \mathbf{W}_t \mathbf{X}$.

4 Linear System

According to the linear dynamics and observation model, the system of **X** is linear. Hence, we can easily prove its observability and calculate the initial state $\mathbf{X}_0$ given the measurements between [0, T] for some T.